WIENER MODELLING AND PREDICTIVE CONTROL FOR WASTEWATER APPLICATIONS

M. O' Brien, L. Giovanni, A. Ordys, R. Katebi Industrial Control Centre University of Strathclyde 50 George St., Glasgow, G1 1BL, UK E-mail of Corresponding Author: marieobrien@eee.strath.ac.uk

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ABSTRACT

The research presented in this paper aims to demonstrate the application of predictive control to an integrated wastewater system with the use of the wiener modeling approach. This allows the controlled process, dissolved oxygen, to be considered to be composed of two parts: the linear dynamics, and a static nonlinearity, thus allowing control other than common approaches such as gain-scheduling, or switching, for series of linear controllers. The paper discusses various approaches to the modelling required for control purposes, and the use of wiener modelling for the specific application of integrated waste water control. This paper demonstrates this application and compares with that of another nonlinear approach, fuzzy gainscheduled control.

INTRODUCTION

Linear predictive control methods have been shown to be applicable for limited control of the wastewater treatment process. For the most part, linear approaches have considered only the control of the treatment plants, and not further systems. The linear methodology has been extended in some instances to the nonlinear integrated system by the use of fuzzy gain-scheduling or switching methods. The method presented in this paper shows an alternative approach to modeling the nonlinear system.

The Wiener or Hammerstein approaches to the modeling of a process is to define two components: the static nonlinearity, and the linear dynamics. The two approaches are defined by the position of the static nonlinearity: preceding the linear dynamic block (in wiener modeling) or following the linear dynamics (in Hammerstein modeling). This approach can be used across industry, as shown in pH control by (Kalafatis et al, 1995), and the distillation column control demonstrated in (Bloemen et al, 2001).

The aim of integrated wastewater control is to maintain river quality, with respect to changes in sewer quality and also the processes within the treatment plant. Integrated control of the dissolved oxygen, and ammonia, processes are demonstrated in (O' Brien, 2005a, 2005b) with the use of fuzzy control. Integrated control was shown for dissolved oxygen in (Rauch et al, 1998) in respect to combined sewer overflow reduction. The approach in this paper therefore does not consider optimisation of the combined sewer overflows, but instead concentrates on the treatment plant processes.

This paper aims to demonstrate the advantage of implementing model based control with the use of a wiener model of the process dynamics. The objectives of the wastewater control are detailed in the next section. Various approaches are discussed within the topic of modelling for control purposes, in the second section. The use of wiener modelling is detailed in the penultimate section, whilst the paper concludes with an application to a nonlinear integrated wastewater treatment model, with the intention of controlling the concentration of dissolved oxygen in the river.

WASTEWATER TREATMENT CONTROL

There are many advantages in the use of predictive control for water quality regulation, in meeting requirements for the both the effluent, and also compliance for the integrated system. This control approach can also be of advantage for the plant operator, because of the intuitive aspect of the predictive approach and the possibility of its use with existing plant sensors, actuators and control loops.

In particular the use of fuzzy methods in predictive control can allow for operator cooperation in the design. The intuitive nature of model predictive control can be seen in the ease of tuning, the possibility of multivariable control, and the simplicity of constraint handling. These aspects result in a control methodology well suited to the regulated water treatment industry.

The existing control technologies in the wastewater industry have concentrated mostly on low-level control strategies within the treatment plant, such as Proportional-Integral (PI) control. Some advanced control methodologies of neural networking and fuzzy logic have been applied in tuning PID controllers by (Rodrigo et al, 1999) in the control of an activated sludge process. The method of model predictive control has, however, been applied within the area of wastewater treatment, concentrating on the optimisation of the treatment plant itself, for example (Alex et al, 2002).

Further research (Lindberg, 1996) has demonstrated the application of dissolved oxygen control in an activated sludge process, with knowledge of the process transfer function in the design of a nonlinear controller. A similar approach is used in this paper, where the process dynamics are instead determined by a wiener model approach for the process, and the control is implemented on the nonlinear integrated system.

The objective of the control demonstrated is to maintain dissolved oxygen levels in the receiving waters in the presence of combined sewer overflows. This predictive approach has been applied by (Grochowski et al, 2004) in which softly switched predictive control is used to alternate between appropriate controllers, involving the use of a series of linear controllers appropriate to certain linear ranges of operation. The wiener model designed here however models the nonlinear behaviour of the system within one model, allowing for the use of a single controller over the operating range.

MODELS FOR CONTROL

Mathematical models, abstractions of real life systems, are of great importance in the understanding of system behavior. From a control engineering point of view it is of great interest how to choose system inputs, such that desired control objectives are met. The intended application of a model, in our case control, will impose restrictions on its structure and complexity. There is a trade-off, since models that can be analyzed and dealt with within a control scheme are often inaccurate, and models that reflect system behavior more accurately are often too complex to analyze and deal with within a control scheme.

Models that relate observed variables (inputs and outputs) by means of a state-space description are common in systems and control theory. According to such a model, at each time instant, the state variables summarize all of the information needed in order to predict together with the future inputs, the future evolution of the system in time. The dynamics of a large class of nonlinear dynamical systems may be cast by a set of nonlinear first order differential equations together with a nonlinear algebraic output equation as follows:

$$\dot{x} = f(x, u, d) \quad x(0) = x_0$$

$$y = h(x, u, d)$$
(1)

with the state $x \in X \subseteq \mathbb{R}^n$, the manipulated input $u \in U \subseteq \mathbb{R}^m$, and the output $y \in Y \subseteq \mathbb{R}^p$.

It is clear that systems as described by (1) are more general than their linear time invariant (LTI) counterparts

$$\dot{x} = Ax + B_u u + B_d d \quad x(0) = x_0$$

$$y = Cx + Du$$
(2)

However a large part of the control literature is devoted to linear systems and many linear system theoretic properties and control problems have been satisfactory dealt with in the literature (Sontag 1998). There are two main reasons for studying linear systems for the purpose of control. Firstly, linear systems are parametrized, and system properties can be revealed easily by analysis of its representation (A, B, C, D). Furthermore, for linear systems the available analysis tools enable strong results on system and control theoretic properties such as stability, controllability, observability, optimality, robustness etc. Secondly, low order approximations are in many cases sufficient to characterize the local behavior of the nonlinear system. This means that often analysis based on linearizations reveals properties of the system locally, and designs based on linearizations often work locally for the original system (Sontag 1998).

This "linearization principle" restricts the applicability of the linear model since desirable and expected behavior of the system can only be guaranteed for operating conditions that are close to the point of linearization. This fact, together with the problem of characterizing system properties, for which the analysis based on its linearization fails, motivated researchers to study nonlinear models, with the intention to derive stronger results, (Nijmeijer and van der Schaft, 1990), (Sepulchre, 1997). Much research effort is directed towards that goal, and since it is not just a matter of extending the linear theory, new concepts are needed.

Instead of studying the general nonlinear model (1) to enlarge the applicability compared to the linear model we will follow another approach based on the linearization principle mentioned above. The idea behind the approach is simple and several researchers and application oriented engineers have come up with similar ideas, confer (Murray-Smith and Johansen, 1997), (Takagi and Sugeno, 1985).

The basic idea is to linearise (1) at several specific operating conditions. The operating conditions are chosen in such a way that the corresponding linear models reflect the qualitatively different behavior in these operating regimes. The operating regimes cover the full operating space. To mimic the behavior of the general nonlinear model, the different linear models obtained for different operating conditions are scheduled in the operating-space, as behavior changes qualitatively under varying operating conditions. Scheduling in this case means forming convex combinations of locally valid linear models. In this way a more global model is obtained that approximately reflects the behavior of the general model for the full range of operation. The resulting model is represented by a finite number N_m of linear models $\{A_i, B_i, a_i, C_i, D_i, c_i\}$ $i=1, ..., N_m$ together with corresponding scheduling functions $\{w_i(x, u, d)\}$ as follows

$$\dot{x} = \sum_{i=1}^{N_m} w_i(u, x, d) \{ Ax + B_u u + B_d d + a_i \}$$

$$y = \sum_{i=1}^{N_m} w_i(u, x, d) \{ Cx + Du + c_i \}$$
(3)

with

$$\sum_{i=1}^{N_m} w_i(x, u, d) = 1, \quad i = 1, \dots, N_m, \forall (x, u) \in X \times U$$

$$w_i(x, u, d) \ge 0 \tag{4}$$

These nonlinear models frequently occur in literature and although of an equivalent mathematical structure they are given different names such as fuzzy models (Sugeno and Kang, 1988; Wang and Mendel, 1992), Hammerstein-Wiener models (Fruzzetti, Palazoglu and Mac Donald, 1997; Norquay, Palazoglu and Romagnoli, 1998; Lussón Cervantes, Agamennoni and Figueroa, 2003), multiple-models (Murray-Smith and Johansen, 1997) or local model networks (Johansen and Foss, The model structure has several desirable 1993). attributes that we would like to exploit. First of all, the model class is rich since a large class of nonlinear systems can be approximated arbitrarily close with the proposed model structure (Wang and Mendel, 1992; Johansen and Foss, 1993). Secondly, the model is interpretable on the basis of regime decomposition (Sugeno and Kang, 1988; Johansen and Foss, 1993). Also the model has an a priori fixed structure, which shows similarities with linear systems. The obtained model is called a Polytopic linear model (PLM) since the model consists of a set of representations of linear models that define a polytope in the models parameterspace.

As previously mentioned, representing a system with an accurate mathematical description of its dynamical properties that is also simple to use for the intended application at hand, results in conflicting demands. On one hand, general nonlinear models can be very accurate but as a result of their complexity difficult to build from first principles and data. Furthermore, these models are difficult to analyze and to apply in model based control schemes. On the other hand LTI systems are well understood, but are at most locally valid in representing an actual system, i.e. incorrect conclusions may be drawn from it, resulting in an unsuccessful control strategy. In the ideal situation, one would like to build a model that is as accurate as the most detailed mathematical model and as simple as a LTI system in the sense as described before. This situation is sketched in Figure 1.



Figure 1. Compromise between the conflicting demands, accuracy and complexity.

The ideal model as indicated in this figure does of course not exist. The choices for a general nonlinear model or a LTI model to represent the system for the intended application in mind are just two possibilities out of numerous possible compromises between complexity and accuracy. The PLM is an alternative compromise between these two conflicting demands that lies within the shaded region in Figure 1. The intended application of the model, in our case control, imposes demands on the model to be selected. The choice for a fixed and flexible model structure that locally shows similarities with LTI systems hopefully enlarges the possibility to develop systems and control theory, and model building methodologies. This hypothesis motivates to explore the suitability of the PLM as a candidate model structure for model based control application. Suitability involves interpretability, representation capacity, and the analysis and controller synthesis abilities as explored in this thesis. As a consequence, it is to be expected that as a result of further research, the distance between the ideal model and the PLM will decrease further in the future. This will probably increase the number of applications for which the PLM is the minimizer of the distance between the ideal model and a member of the set of all candidate model structures.

WIENER MODELLING

When the gain of the system changes with operating point while the dynamics remain constant, the PLM (1) reduces to a Wiener model. This model is defined as being composed of two components: the static nonlinearity, and the linear dynamics, which are both determined from step tests on the system. The static nonlinearity of the system results from the variance of the system steady state gain over the nonlinear range. The linear dynamics, in this case, can be approximated effectively with a simple linear transfer function.

In this application, the static nonlinearity can be modeled as a simple function as follows:

$$v(k) = f(u(k), d(k).u(k))$$
(5)

where C is the steady state gain of the system, whilst the linear dynamics modeled by a transfer function can be defined equivalently by a state space function:

$$x(k | k+1) = Ax(k | k) + B.v(k | k)$$

y(k | k) = Cx(k | k) + D.v(k | k) (6)

The wiener model can then be written in the form of an input-dependant state space system, where the states of the model are defined by the above models of (5) and (6)

$$x(k | k+1) = Ax(k | k) + B(u, d).u(k | k)$$

$$y(k | k) = Cx(k | k) + D(u, d).u(k | k)$$
(7)

where u(k | k) is the vector $\begin{bmatrix} u_1(k | k) & u_2(k | k) \end{bmatrix}$ and B(u, d) and D(u, d) are dependent on equation (5). This allows the model to be updated at each sampling instant, allowing a In order to incorporate integral action within the controller, the model as shown in the above equation is considered to be defined with respect to input increments:

$$x_{Inc} \left(k \mid k+1 \right) = \begin{bmatrix} A B(u) \\ 0 & I \end{bmatrix} x_{inc} \left(k \mid k \right) + \begin{bmatrix} B(u) \\ I \end{bmatrix} \Delta u(k \mid k)$$
$$y \left(k \mid k \right) = \begin{bmatrix} C D(u) \end{bmatrix} x_{inc} \left(k \mid k \right)$$
(8)

where x_{inc} is defined as an adjusted state vector:

$$x_{inc} = \begin{bmatrix} x(k \mid k) \\ u(k \mid k - 1) \end{bmatrix}$$
(9)

In more concise notation, the model can be seen as:

$$x_{Inc} \left(k \mid k+1 \right) = \tilde{A} \cdot x_{inc} \left(k \mid k \right) + \tilde{B} \cdot \Delta u(k \mid k)$$

$$y \left(k \mid k \right) = \tilde{C} \cdot x_{inc} \left(k \mid k \right)$$
 (10)

where \widetilde{A} , \widetilde{B} and \widetilde{C} are defined by equation (9). The function of an appropriate nonlinear controller consists of the following steps, at each sample instant:

- 1. the estimation of the system states according to the above state space model, in this case using the kalman approach
- 2. The prediction of future behaviour, found by iterating the above model over the prediction horizon, and using the states calculated by the kalman estimator
- 3. The calculation of the optimal control for the plant, according to a user-specified cost function

4. The application of the first element of this control and the return to Step 1 at the next sample instant.

The state estimation is found using the approach of an extended kalman filter, where the state space equations are updated at each sampling instant. The kalman gain is determined from these state space equations, and thus is also updated. The predictions are found using the predictive approach as demonstrated by Krauss et al (1994), and the control actions are calculated to minimize a user-specified cost function.

APPLICATION

The process under study was that of an integrated wastewater system, composed of a sewer network, a treatment plant and the receiving waters. The process model used to describe this system was that produced by Camilleri et al (2004). The treatment plant model was that of the ASM2d (IWA, 2000), which was used to model an anoxic-aerobic process within the plant, whilst the Takàcs' model (Takács et al., 1991) was used to represent the secondary clarifier. The sewer model consisted of a catchment model, the unit hydrograph of (Sherman, 1932), and a detention tank simulated as a CFSTR (Continuous Flow Stirred Tank Reactor). Hydraulic and biological or chemical processes were assumed not to occur within the detention tank. The model of the receiving wasters was based on that demonstrated by the QUAL2E model (Brown, L. C. and Barnwell Jr., T. O. (1987)). The river was represented by a series of segments with the same properties, modelled as above by a CFSTR. The mass transport equations described in (Camilleri et al, 2004) model the behaviour of the fractions in the river.

The wiener model is developed by performing step tests on the nonlinear system, over the flow range. The data is divided into two components: the component that changes over the flow range (the steady state gain) and the component which does not change over the flow range (the transfer function of the step response). The first of these was determined by calculating the steady



Figure 2. Normalised step responses

Π4

state gain of the system at different flow values, as is shown in Figure 2.

The transfer function was determined using subspace identification techniques with the normalised step responses.

$$G(s) = \frac{0.45s + 1}{s + 1} \tag{11}$$

With the transfer function of the dissolved oxygen dynamics, and the nonlinear gain function, the wiener model can be built. The nonlinear gain was determined by interpolation and is shown in Figure 3, where it is compared to a linear approximation. Figure 4 shows the comparison between the responses of the system and the wiener model. The full wiener model is a suitable approximation of the nonlinear dissolved oxygen dynamic within this case study.

The storm event presented here for the wiener dissolved oxygen control is of duration 0.33h at an intensity of 30mm/h at time 2 days, the controller was tuned to a set point of 7.6g/m3. It is compared in Figure 5 with the response of the fuzzy MPC designed in (O' Brien et al, 2005b). It can be seen that the Wiener MPC



Figure 4. Full Wiener Model versus real system for nonlinear storm event



Figure 5. Wiener MPC versus Fuzzy MPC



Figure 6. Wiener versus Fuzzy: control action

outperforms the fuzzy predictive controller in setpoint tracking, and also in some instances in disturbance rejection. The wiener MPC can be seen at points to underperform the fuzzy MPC in disturbance rejection, however this is with the advantage of a less oscillatory response and a quicker return to steady state conditions.

Another advantage of the wiener is the smoother control action, compared with the oscillatory response of the fuzzy MPC, as can be seen in Figure 6. The Wiener model also has the advantage of being determined from simple step responses as opposed to more complex system identification methods such as fuzzy logic or neural networks. The fuzzy model uses several linear models, and thus requires tuning for each, whilst the Wiener model uses only one system model, thus reducing computational costs and tuning required.

CONCLUSION

The research presented in this paper detailed the approaches of modelling for a control purpose, with particular application to an integrated wastewater system. The objective of this control was to maintain dissolved oxygen levels in the receiving waters, and this was obtained through the application of wiener model predictive control to the process. The results shown were compared with that of previous work in fuzzy gain-scheduled control.

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AUTHOR BIOGRAPHIES

- MARIE O' BRIEN was born in Cork, Ireland and went to University College Cork, where she studied Electrical Engineering and Microelectronics and obtained her degree in 2002. She worked in the area of control in the pharmaceutical industry before moving in 2003 to the University of Strathclyde where she is now studying for her doctorate.
- **DR. REZA KATEBI** obtained the degrees of BSc (Hons), MSc and his PhD at UMIST, Manchester. He continued with research work in the Industrial Control Unit and Industrial Systems and Control Ltd., Glasgow, and joined the Department of Electronic and Electrical Engineering. He is now Senior Lecturer in the Control of Manufacturing Systems working in the Industrial Control Centre. Email: r.katebi@eee.strath.ac.uk
- **DR. ANDRZEJ ORDYS** is the British Energy Lecturer in Control Systems and the Technical Manager of the Industrial Control Centre at the University of Strathclyde. He is Member of IEE and Chartered Engineer, and Member of IEEE and of IAPR. He is Editor of Book Reviews in the International Journal of Adaptive Control and Signal Processing. Email: a.ordys@eee.strath.ac.uk
- **DR. LEONARDO GIOVANINI** obtained the BSc (Honors) degree in Electronic Engineering from the Universidad Tecnologica Nacional in 1995, and the

Ph.D. degree in Engineering Science from Universidad Nacional del Litoral, School of Water Resource in 2000. Since then he has been Lecturer in the Dept. of Electronic Engineering at Universidad Tecnologica Nacional until 2002 when he joined the Industrial Control Center. He is now Senior Research Fellow and Centre Technical Manager. Email:

leonardo.giovanini@eee.strath.ac.uk