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Short-time multifractal analysis: application to biological signals

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Abstract. Some signals obtained from biological systems evince a great complexity. Recently, new tools which allow the extraction of information from them have been proposed. In particular, multifractal analysis gives a quantification of the degree and distribution of irregularities in a signal. A possible approach for this analysis is the one based on wavelet leaders. In this work, the use of wavelet leader based multifractal analysis in short-time windows is proposed in order to analyze the evolution of the multifractal behavior of biological signals. In particular, applications of this technique to the detection of ischemic episodes in heart rate variability signals and to voice activity detection are examined. It is shown that the study of the time evolution of indexes obtained with the proposed new method gives useful information hidden in the signals.

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1. Introduction

Some biological systems show a great complexity. Signals from these systems are generally nonlinear and nonstationary, and show complex and irregular fluctuations even under resting conditions. This fact defies the traditional concept of *homeostasis*, according to which healthy biological systems are in equilibrium and self-regulated to reduce variability [1]. On the contrary, these systems show erratic fluctuations resembling those of systems operating far from equilibrium. Moreover, these fluctuations contain hidden information about the control systems operating in this non equilibrium state [2].

In the 1990s, tools from statistical physics were first used to cope with this complexity in order to extract this hidden information coded in the fluctuations of time series (TS). In particular, the multifractal formalism (MFF) allows to study scaling phenomena and long-range correlations in TS, providing a quantification of its singularities distribution. A statistical approach based on the continuous wavelet transform gave rise to a unified multifractal description [3]. Variants were proposed, based on the wavelet transform modulus maxima (WTMM) [4], the discrete wavelet transform (DWT) [5] and, more recently, the wavelet leaders (WLs) [6],[7].

Multifractal analysis has been successfully applied to the analysis of numerous biological signals: DNA sequences [4], heart rate variability [5], EEG [9], human gait [10] and speech [11], amongst others.

In the present work, we propose the application of WLs based multifractal analysis in short-time windows in order to analyze the evolution of the multifractal behavior of the signal. The selection of the window length is discussed using artificial signals. Two applications of this technique to real

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biological signals are presented: detection of ischemic episodes in HRV signals and voice activity detection (VAD.)

2. Materials and methods

In this section a brief review of the tools used in the present work is given. The definitions of Hölder exponent and the singularity spectrum are presented. The WLs and the corresponding multifractal formalism are described, following [7]. Finally the short-time multifractal analysis proposed in the present work is presented. The signals used for the experiment are described in the last subsection.

2.1. Hölder exponent and singularity spectrum

Given a point $t_0 \in \mathbb{R}$ and a real constant $\alpha \geq 0$, a function $f : \mathbb{R} \to \mathbb{R}$ is said to be $C^{\alpha}(t_0)$ if there exists a constant K > 0 and a polynomial P_{t_0} of degree less than α, such that $|f(t) - P_{t_0}(t)| \leq C\,|t - t_0|^{\alpha}$. The Hölder exponent $h_f(t_0)$ of f in t_0 is defined as $h_f(t_0) = \sup\{\alpha : f \in C^{\alpha}(t_0)\}$ [6]. It measures the local regularity of f in t_0. Small (close to 0) values of the Hölder exponent denote strong and sharp singularities, whereas large values denote smooth ones.

When analyzing a singular almost everywhere signal, it might be useful to know the distribution of the singularities with a given Hölder exponent. This information is provided by the singularity (or multifractal) spectrum (SS): it measures the *amount* of singularities with a given Hölder exponent. Formally, it is defined as the Hausdorff dimension of the set of points whose Hölder exponent is h: $D(h) = \dim_{\mathrm{H}}\left\{t \in \mathbb{R} : h_f(t) = h\right\}$.

2.2. Multifractal formalism

The determination of the SS of a signal is important to analyze the singularities present in a given signal. However, it can not be numerically determined from its definition because of the limitations imposed by finite resolution and the sampling of signals [7]. In order to cope with this situation, a *multifractal formalism* (MFF) was introduced. The MFF provides an alternative way to compute the SS based on easily computable quantities: the structure functions (SFs). As already mentioned, a new MFF, based on WLs, was recently proposed [7]. This approach solves many of the drawbacks of previous methods. Its main features are the fact of being based on the DWT, which allows an efficient implementation in terms of filter banks, and its numerical stability.

2.3. Wavelet leaders

Let ψ_0 be a mother wavelet with compact support and a number $N \geq 1$ of vanishing moments. Let $\left\{\psi_{j,k}(t) = 2^{-j}\psi_0(2^{-j}t - k), j \in \mathbb{N}, k \in \mathbb{N}\right\}$ be the orthonormal basis of $L^2(\mathbb{R})$ formed by templates of ψ_0 dilated by scales 2^j and translated to time positions 2^jk. The DWT coefficients of f are obtained by means of the inner products: $d_f(j,k) = \int_{\mathbb{R}} f(t) 2^{-j}\psi_0(2^{-j}t - k)\mathrm{d}t$.

A special notation is introduced for dyadic intervals. Let $\lambda = \lambda_{j,k} = \left[k2^j, (k+1)2^j\right)$ so that $d_\lambda \equiv d_f(j,k)$. Finally, let 3λ be the union of the dyadic interval λ and its two adjacent intervals: $3\lambda_{j,k} = \lambda_{j,k-1} \cup \lambda_{j,k} \cup \lambda_{j,k+1}$.

The WLs are defined as:

$$L_f(j,k) \equiv L_\lambda = \sup_{\lambda' \subset 3\lambda} \left|d_{\lambda'}\right|. \tag{1}$$

In other words, the computation of the WLs for a given time and scale involves the search of the greatest wavelet coefficient in a narrow time neighborhood, for all finer scales.

2.4. Wavelet leader based multifractal formalism

SFs are computed from WLs:

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$$S_L(q,j)=\frac{1}{n_j}\sum_{k=1}^{n_j}\left|L_f(j,k)\right|^q. \quad (2)$$

If the signal f shows some form of self-similarity, SFs decay as power laws of the scales. The exponents of these power laws are called *scaling exponents* (SEs):

$$\varsigma_L(q)=\liminf_{j\to 0}\left(\frac{\log_2 S_L(q,j)}{j}\right). \quad (3)$$

In practice, SEs are obtained as the slopes of the linear regressions of SFs *vs.* scales, in a logarithmic graph.

Finally, the SS can be obtained from the SEs via a Legendre transform (LT):

$$D(h)=\inf_{q\neq 0}\left(1+qh-\varsigma_L(q)\right). \quad (4)$$

2.5. Short-time multifractal analysis

MFA of a signal provides global information concerning its singularities distribution. However, in some cases local information is required, e.g. in the detection of transient states that cause changes in the singular behavior of signals. With this in mind, in the present work we propose the application of MFA in short-time windows to analyze the evolution of the multifractal behavior of signals. Avoiding the study of the whole SS, we propose to use multifractal indexes to summarize the information in a few coefficients and to analyze their evolutions.

As first index we propose to use the dominant regularity h_{max}, which is defined as the Hölder exponent for which the spectrum has its maximum: $D(h_{\max})>D(h),\forall h\neq h_{\max}$. Small (close to zero) values of this index denote signals with predominantly sharp singularities, whereas large values correspond to signals that mainly have smooth singularities.

As second index we consider the spectrum's width at 80% of its height, W_{80}. It measures the *multifractality* of the signal. Given $h_1<h_{\max}<h_2$ such that $D(h_1)=0,8\,D(h_{\max})$, $h_1<h_{\max}$, and $D(h_2)=0,8\,D(h_{\max})$, $h_2>h_{\max}$, then $W_{80}=h_2-h_1$. This last index measures the local regularity distribution. Large values reflect that the signal has singularities of very different Hölder exponent (wide spectrum). Small values reflect signals with singularities that have similar Hölder exponents (narrow spectrum). When all the singularities of a signal have the same Hölder exponent, the SEs is a line with slope equal to this Hölder exponent and the SS is a point. These kinds of signals are called monofractals.

With these two indexes, we propose to perform a short-time multifractal analysis, meaning to compute each index in a sliding window applied to the signal. The values obtained in each window are assigned to its middle point.

2.6. Signals and databases

We have considered a synthetic signal and two types of real signals: RR time series and speech signals.

In order to analyze the influence of the window length, artificial signals formed by the concatenation of three segments of fractional Brownian motion (fBm) with different Hurst parameters were used: $s=(s_1|s_2|s_3)$. fBm is monofractal: it is singular in almost every point and all singularities have the same Hölder exponent, which is equal to the Hurst parameter [7]. Segments s_1 and s_3 consisted in 2^{18} samples of fBm with the same Hurst parameter H_1. Segment s_2 consisted in 2^{19} samples of fBm with Hurst parameter H_2 (figure 1.a.) Different combinations of H_1 and H_2 and 100 realizations of each combination were analyzed.

For the analysis of HRV, the European ST-T database (ESDB) [12] was used. It comprises two-channel ECG records, two hours long each, of patients suffering from myocardial ischemia. Each record contains at least one ischemic episode. Records are digitized with a sampling frequency of 250 Hz. For each record, the ESDB has one annotations file with information provided by experts. It includes the position of each beat, the beginning and end of each ischemic episode (obtained from the

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deviation of the ST segment and T wave of the ECG) and the characteristics of each beat (normal, ectopic, noisy, etc).

From the annotations file, RR interval series were obtained considering the difference between two consecutive beats. All intervals corresponding to abnormal beats were eliminated. This procedure prevents the inclusion of artifacts, avoiding in this way the need of a filtering stage. Finally, the signals were uniformly interpolated with cubic splines and a sampling frequency of 4 Hz, given that an uniformly sampled signal is required for the DWT computation.

For the study of voice activity detection, the database BEPPA [13] was used. This database comprises a set of words and sentences of argentinean Spanish. Records are digitalized with a sampling frequency of 48 kHz. In this preliminary work, only the subset of monosyllabic words MW02 was used. This list comprises 10 monosyllabic words.

3. Results

3.1. Influence of window´s length

In order to study the influence of the window´s length, we consider the synthetic signals described in Sec. 2.6. Given that the signal to be analyzed is a concatenation of monofractals, almost constant values of h_{max}, close to the corresponding Hurst parameter, and small values of W_{80} are expected for each of the time segments corresponding to the concatenated signals.

Figure 1 shows one realization, with H_1=0.3 and H_2=0.7 (a), and the evolutions of the multifractal indexes for two window lengths: 2^{15} samples (b-c) and 2^{11} samples (d-e).

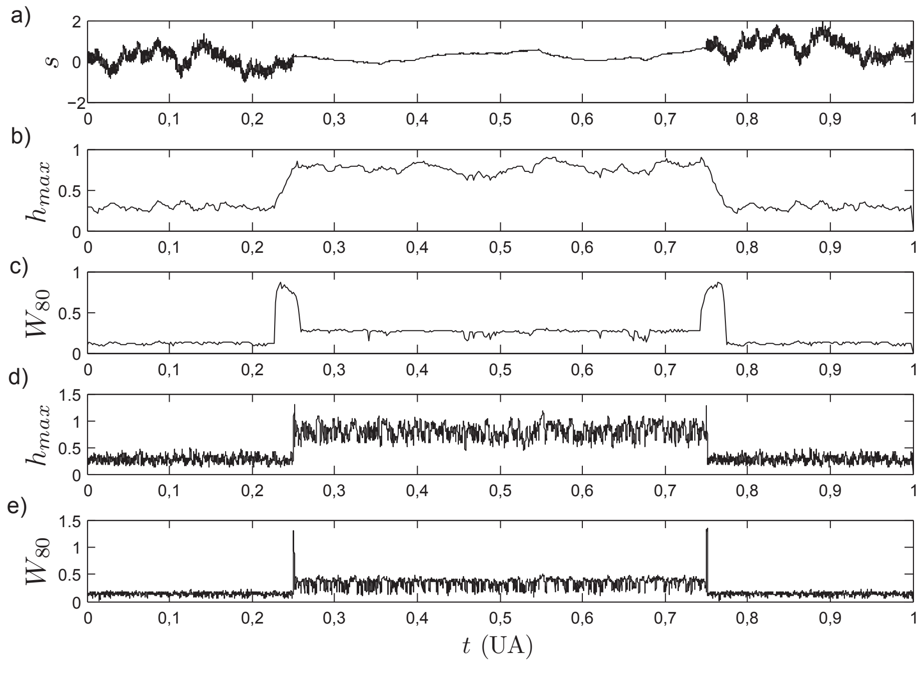

Figure 1. One realization of the artificial signals described in section 2.6, with H_1=0.3 and H_2=0.7 (a) and time evolutions of h_{max} and W_{80} for window´s lengths of 2^{15} samples (b-c) and 2^{11} samples (d-e).

It can be appreciated that when the window length is large (2^{15} samples), h_{max} assumes values close to the expected value. On the contrary, when a small window (2^{11} samples) is used, the evolution shows wider fluctuations around it. A similar phenomenon can be observed in the evolutions of W_{80}. This loss of precision when a small window is used is due to the fact that the number of samples limits the number of scales that can be obtained to perform the DWT. On one hand, this causes a loss of

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precision in the estimation of the WLs, which requires the search of the largest wavelet coefficient in a wide range of scales. On the other hand, the range of scales over which the linear regressions are performed to obtain the SEs from the SFs is also reduced, causing an additional loss of precision.

The means and standard deviations (*std*), for both window´s lengths considered and 100 realizations of segments s_1 and s_2, are shown in figure 2 (a) and (b), respectively. Points corresponding to windows of 2^{15} and 2^{11} samples are shown in black and gray, respectively. Dots and crosses are used for segments with Hurst parameter H_1 and H_2, respectively. It can be seen that the mean of h_{max} is close to the Hurst parameter of the segment for both window sizes, while the *std* is larger when using the short windows. Similar results have been obtained in a more extensive study performed by the authors, using different values of H_1 and H_2 [14]. These experiments illustrate the loss of precision in the estimation of the multifractal indexes when using short windows.

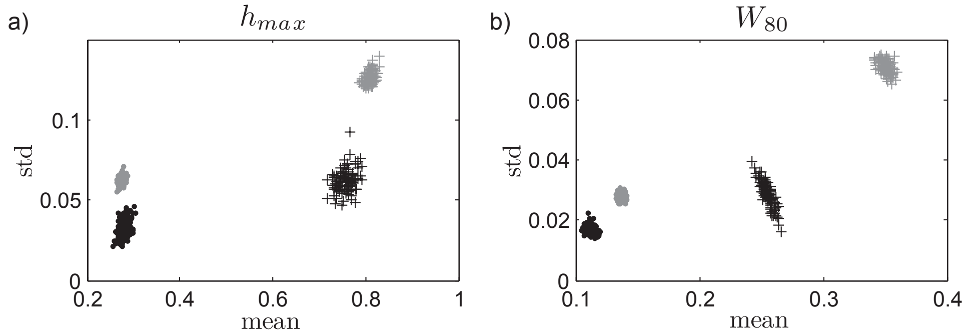

Figure 2. Mean *vs.* *std* of indexes h_{max} (a) and W_{80} (b) for 100 realizations of the artificial signals described in the text. Dots: segments with Hurst parameter H_1. Crosses: segments with Hurst parameter H_2. Black: 2^{15} samples long window. Gray: 2^{11} samples long window.

The use of short windows allows for a better temporal resolution. This can be seen clearly in figure 1 (c) and (e), for the index W_{80}. The transitions between segments cause A_{80} to show large values on a narrow region when the narrow window is used (c,) and on a wide one when the large window is used (e).

These experiments suggest that the window length is a parameter that should be chosen carefully and as a tradeoff between temporal resolution and accuracy in the estimation of the SS.

3.2. Analysis of ischemic episodes

It was shown in [8] that multifractal analysis of HRV conveys useful information concerning the dynamic changes that take place during myocardial ischemia. The SEs and SS corresponding to a normal zone and an ischemic zone of record e0154 of the ESDB are shown in figure 3 (a) and (b), respectively. It can be seen that, in the presence of ischemia, the SS becomes wider and the predominant regularity h_{max} moves to higher values (singularities become smoother). This information can also be seen in the SEs. Indeed, the SEs corresponding to the ischemic zone shows a greater curvature and its least square approximation line shows a larger slope (smoother predominant local regularity) than those of the normal zone.

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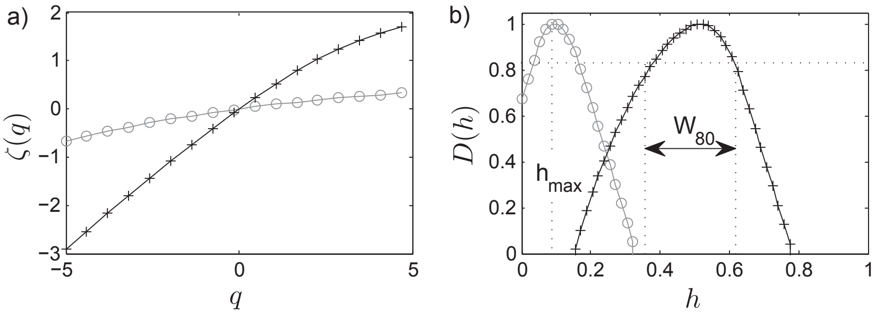

Figure 3. Scaling exponents $\varsigma_L(q)$ (a) and singularity spectrum $D(h)$ (b) corresponding to a normal zone (gray circles) and an ischemic zone (black crosses). Also, the indexes h_{max} and W_{80} are illustrated.

In the short-time analysis of RR signals, 2^{11} samples (512 s) long windows and 128 samples time shifts were used. Figure 4 shows a RR signal, with an ischemic episode indicated by vertical gray lines (a), and the time evolutions of h_{max} (b) and W_{80} (c). It can be appreciated that, when approaching the ischemic zone, both indices show a sharp rise and that after the episode they move down to values similar to those obtained before the ischemic episode. While analyzing these evolutions, it must be taken into consideration that myocardial ischemia entails a series of alterations in which metabolic changes precede hemodynamic changes, which in turn precede electrocardiographic changes. As a consequence, the ischemic episode is reflected in the RR signal beyond the bounds indicated by the marks obtained from the ECG.

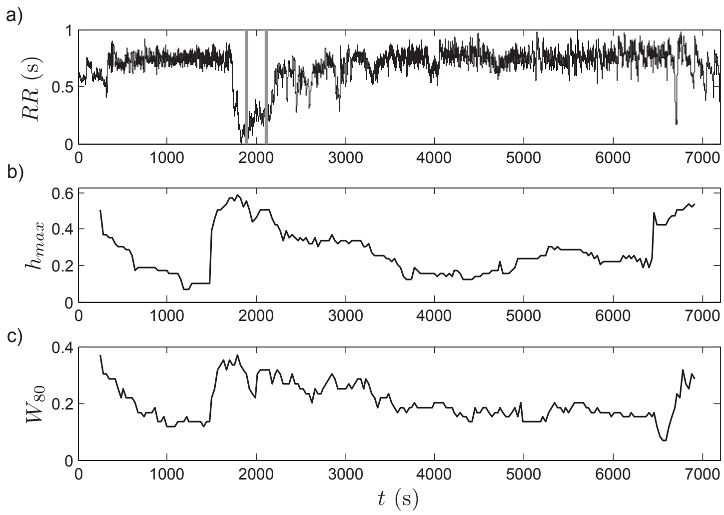

Figure 4. RR signal (a) with an ischemic episode indicated by vertical gray lines and the windowed time evolutions of h_{max} (b) and W_{80} (c).

3.3. Voice activity detection

A fractal analysis of speech signals was proposed in [11] with the objective of obtaining useful information for voice activity detection and phoneme recognition applications. This analysis was justified by turbulence phenomena in the air flow during speech production and other nonlinear dynamics of the human vocal-tract system. This work was based on the concept of fractal dimension,

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which characterizes the complexity and irregularity of a signal using, unlike multifractal analysis, only one index.

This approach is extended in the present work to a multifractal analysis applied to VAD. The hypothesis that the regularity of voice signals is different than the one corresponding to the background noise is assumed.

A short-time multifractal analysis was performed, using 2^{11} samples (43 s) long windows and 256 samples time shifts.

As a comparison, the time evolution of the zero-crossing rate (*ZC*), a standard index in VAD, was computed. The same window size and time shift than in the multifractal approach were used.

Figure 5 shows the sonogram of the Spanish word “flan” [flán] (a) and the time evolutions of the multifractal indexes and ZC (b-d). The evolutions of the multifractal indexes are shown in semi-log graphs to highlight structures that take place when the indexes’ values are small. It can be seen that h_{max} has a similar behavior than *ZC*, whereas W_{80} fails to properly detect the final portion of the word. If the interval between 0.3 s and 0.5 s, corresponding to the non-sonorant fricative phoneme /f/, is carefully examined, it can be observed that in figure 5 (d) *ZC* fails to recognize this phoneme whereas figure 5 (b-c) indicate that both multifractal indexes display a small rise in this region . On the other hand, both h_{max} and W_{80} show a small peak around 0.3 s (indicated by the circles and arrows in figure 5, (b-c)), which is coincident with the beginning of the phoneme. During all the phoneme’s length, W_{80} remains with a value slightly larger than the one corresponding to the noise that preceeds it..

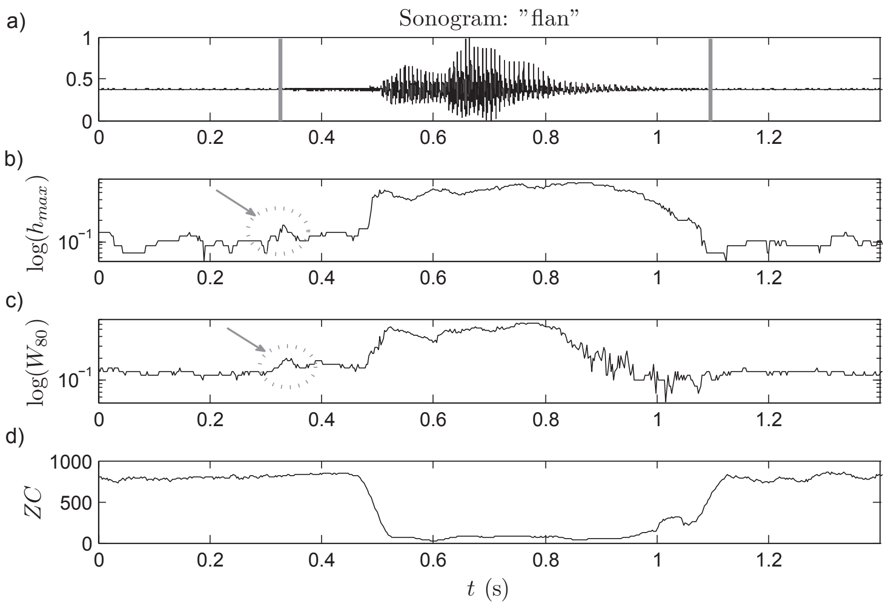

Figure 5. Sonogram of the Spanish word “flan” (a), with a manual segmentation performed by the authors indicated by vertical gray lines, and windowed time evolutions of indexes h_{max}, W_{80} (in a semi-log graph) and the zero-crossing rate *ZC* (b, c and d, respectively).

These experiments suggest that the joint use of the proposed multifractal indexes might provide proper voice activity detection, even for the fricative phonemes where the zero-crossing rate is similar to the one of the background noise. This is an observation that should be taken into consideration in VAD in Spanish language, given the importance of this type of phonemes in this language.

4. Conclusions

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In the present work we have proposed to use multifractal analysis based in wavelet leaders, in short-time sliding windows to analyze biological signals. The application of this technique to two different kinds of real biological signals was examined. Results suggest that the study of the windowed time evolution of indexes derived from the SS is useful for the detection of transient alterations that cause changes in the regularity of the signal.

The effect of the window´s lenght has been discussed with synthetic signals. . This parameter should always be chosen carefully as a tradeoff between temporal resolution and accuracy in the estimation of the SS.

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