Comparison of on-line wavelet analysis and reconstruction: with application to ECG

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***Abstract*—The wavelet transform is a consolidated tool for signal analysis. However, it has limitations for processing on-line and long duration signals. In this paper we applied three alternatives for on-line processing based in the wavelet transform which are suitable for biomedical signals. The methods for on-line wavelet processing was compared with the off-line DWT as baseline reference. The output for each methods was analyzed in detail by using the mean square error and the normalized maximum amplitude error. We concluded that the best performance is obtain with Hamming windows and overlap-and-add for reconstruction. As an application example, we show good results for the on-line processing of ECG signals.**

***Index Terms*—DWT, reconstruction error, on-line processing.**

I. Introduction

Wavelets have been used in different areas of biomedical signal processing such as feature extraction, compression and noise removal [1]. This tool differ from traditional Fourier analysis in that it localize the information in the time-scale plane, and this is useful for the study of nonstationary signals.

Researchers have used wavelets for the analysis of ECG [2], EEG [3], evoked potentials [4], gastrointestinal movement [5], speech [6] signals, and noise reduction, enhancement and detection of diseases in biomedical images [7]. Moreover signal compression with wavelet is important for applications in telemedicine [8].

Traditionally the wavelet transform (WT) has been used for off-line signal analysis and processing. This is an important limitation for long duration signals or for applications that require real time processing. In this work some methods which are classical signal processing are adapted, to overcame this limitation in the wavelet transform. The proposed methods are windowing with: overlap-and-add, padding with zeros and overlap-and-save, the first one was adapted from the STFT and the rest were adapted from long term filtering.

In the next sections we will detail the methods, first introducing some generalities about wavelet transforms and then presenting the windowing and the reconstruction methods. Next, we compare those methods with respect to the off-line wavelet processing for the whole signal using the mean square error and the normalized maximum amplitude error. As an example application, the methods which produced best results was used for baseline wander removal of the ECG. A final section presents conclusions of the work.

II. Methods

A. Dyadic Wavelet Transform

In this section we will introduce some generalities about wavelet transform. The WT is a time-scale representation that decompose a signal x into a basis of time and scale functions which are dilated and translated versions of a basic function ψ which is called mother wavelet [9]. A dyadic wavelet transform is generated from a dyadic translation and dilatation of the mother function

$$\psi_{j,k}[m] = \frac{1}{2^j}\psi\left[\frac{m-k}{2^j}\right], \tag{1}$$

where j is the decomposition level and k the translation.

A discrete dyadic WT (DWT) can be designed and implemented as a filter bank [9]. In this case one never has to deal directly with the wavelet, only the coefficients of the associated filters are needed. A highpass filter $\bar{\mathbf{g}}$ give the detail coefficients $\mathbf{d}$, and a lowpass filter $\bar{\mathbf{h}}$ give the approximation coefficients $\mathbf{a}$. This operation can also be written in matrix form, using a matrix operator $\mathbf{W}$ that implements the filtering and subsampling at each scale:

$$\mathbf{w} = \mathbf{W}\mathbf{x},$$

where $\mathbf{w}$ represent the DWT coefficients vector.

B. Windowing with overlap and add

The first method for short time analysis and synthesis is called windowing with overlap and add (WOAA). Suppose we have a digital signal x and its windowed and shifted version given by

$$x^{\tau}[m] = \phi[m]x[m+\tau N_s], \tag{2}$$

where $\phi[m]$ is a window (e.g. Hamming, boxcar, etc.) of length N_w, and N_s is the frame shifting for the analysis. Consider the DWT of a signal as an orthonormal transform. Let $\{w[n] : n =$

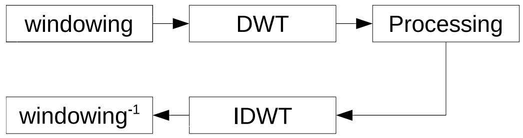

Fig. 1. Diagram of the algorithm

$0, 1, ..., N-1\}$ represent the DWT coefficients, we can write $\mathbf{w}^{\tau} = \mathbf{W}\mathbf{x}^{\tau}$, where $\mathbf{W}$ is a $N_w \times N_w$ matrix defining the DWT.

After applying this transform to each of the frames of our windowed signal, some processing in the wavelet domain can be applied. The processing (Fig. 1) can be, for example, a simple thresholding operation to eliminate some noise or zeroing the coefficients associated with some scale. After processing, we apply an inverse discrete wavelet transform (IDWT) $\mathbf{z}^{\tau} = \mathbf{W}^{T}\mathbf{w}^{\tau}$ to each frame, to obtain the processed windowed signal. Using each reconstructed frame we use the overlap-and-add method [10] to resynthesize the signal,

$$\begin{aligned} z^*[m] &= \sum_{\tau=0}^{L-1} z^{\tau}[m-\tau N_s] \\ &= \sum_{\tau=0}^{L-1} \phi[m-\tau N_s] z[m-\tau N_s+\tau N_s] \\ &= z[m] \sum_{\tau=0}^{L-1} \phi[m-\tau N_s] \\ &= z[m]\Phi[m], \end{aligned} \tag{3}$$

where $z^*[m]$ denotes the sum of all reconstructed frames and $\Phi[m]$ the shifted sum of the windows. From (3) the signal can be reconstructed as $z[m] = \frac{z^*[m]}{\Phi[m]}$.

In this way, the signal can be processed frame by frame, which is a suitable method for on-line processing of long data, where a buffer is filled until the required number of samples are acquired, and then processed, without the need of waiting for full acquisition before starting the analysis.

C. Windowing with overlap and save

The method of windowing with overlap and save (WOAS) consist in adding $M-1$ zeros at the begging of the signal, and a boxcar window of length N_w was used to segment the signal, where M is the length of the wavelet filter. This block is transformed with the DWT and processed. The next block overlap the previous block in $M-1$ samples. For the reconstruction the IDWT is used, discarding the $M-1$ first samples of each block and save the rest as Fig. 2 show.

D. Windowing and padding with zeros

The third proposed method is the windowing and padding with zeros (WPZ). The signal is segmented with a boxcar window of length N_w, without overlap. Then a block of N_w zeros is added at the end of each segment

$$\mathbf{x}^1 = \{x[0], ..., x[N_w-1], 0, ..., 0\}$$

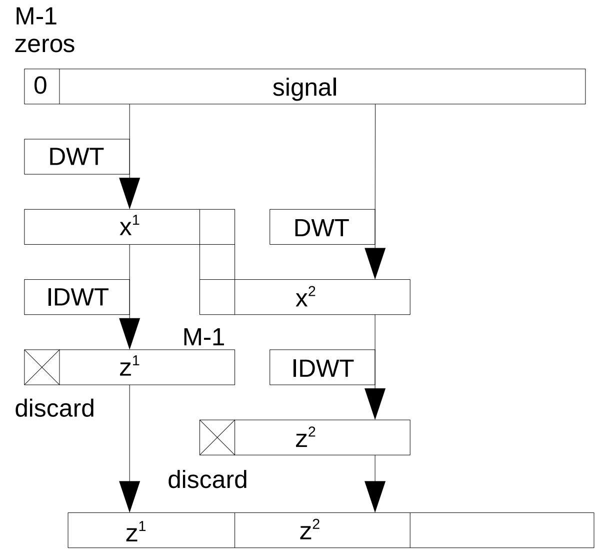

Fig. 2. Diagram of the algorithm windowing with overlap and save

$$\vdots$$

$$\mathbf{x}^j = \{x[(j-1)N_w], ..., x[jN_w-1], 0, ..., 0\}.$$

The DWT is applied to each $\mathbf{x}^j$ and in the wavelet domain the signal can be processed. Note that $\mathbf{x}^j$ has length of $2N_w$. For the reconstruction the IDWT is used to obtain $\mathbf{z}^j$. The last part of $\mathbf{z}^j$ of length N_w, which correspond to the transform of the zero pads, is added to the first part of $\mathbf{z}^{j+1}$ of N_w samples to reduce borders effects.

III. Comparative Study

The experiments consist of transforming a signal using either of the windowing methods, applying some processing to the wavelet coefficients, and reconstructing the signal using the corresponding reconstruction method. Also, the signal is analyzed using the standard off-line method for the wavelet transform of the whole signal using the same decomposition level. The same processing of the wavelet coefficient is applied, and the signal is reconstructed by an IDWT. Then a error measure is applied to quantify the difference of the signals processed by the three short-time methods with respect to the signal processed by the standard DWT.

Two error measurement were used: the mean square error, $e_{MSE} = \frac{1}{P}\sum_{m=1}^{P}(s[m]-\bar{s}[m])^2$, which gives an idea of the error in all the signal, and the normalized maximum amplitude error [11]

$$e_{NMAE} = \frac{max(\mathbf{s}-\bar{\mathbf{s}})}{max(\mathbf{s})-min(\mathbf{s})}, \tag{4}$$

which emphasize the maximum peak of the error instead of average error over all signal. In both definitions, $\mathbf{s}$ is the reference data and $\bar{\mathbf{s}}$ is the processed data.

A first experiment was the analysis of the errors by transformation and reconstruction of a signal, without any processing

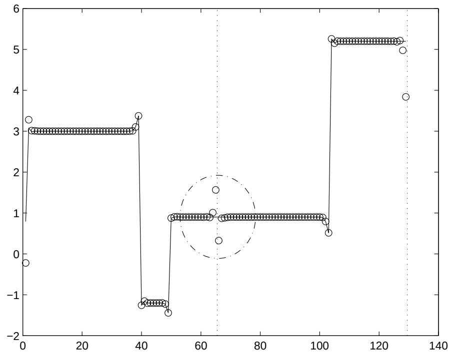

Fig. 3. An extract of Block signal processed by zeroing the detail coefficient at the first scale by both the off-line DWT in solid line and by WPZ in circles with mother wavelet Daubechies 8. In dotted we observed the step of the moving window, and in the circle in dash dot the border effect.

TABLE I
ERROR BETWEEN THE CLASSICAL OFF-LINE DWT PROCESSING WITH RESPECT THE METHODS ANALYZED WITH MOTHER WAVELET DAUBECHIES 8 AND ZEROING THE APPROXIMATION COEFFICIENT IN WAVELET DOMAIN AT LEVEL 8

	Blocks		Doppler	
	e_{MSE}	e_{NMAE}	e_{MSE}	e_{NMAE}
WOAA Hamming, $N_s = 32$	0.2743	0.1392	0.0049	0.1066
WOAA Boxcar, $N_s = N_w$	0.5359	0.2104	0.0271	0.3448
WOAA Boxcar, $N_s = 32$	0.2329	0.1189	0.0067	0.1095
WZP	0.1923	0.2647	0.0109	0.2202
WOAS	0.2712	0.1277	0.0151	0.3050

in the wavelet domain. The e_{NMAE} and e_{MSE} are in the order of 10^{-12} and 10^{-23} respectively, showing that all methods are suitables for reconstruction. The problem is when some modification is applied in the wavelet domain, as an example we can see in Fig. 3, an extract of the Blocks signal [12] processed by zeroing the detail coefficients at the first scale to eliminate the high frequency contents, by both the off-line DWT is in solid line and by WPZ in points. In solid gray we observed the step of the moving window, where we observed a pseudo-gibbs phenomena introduced by the border of the processing window.

Next, the reconstruction of the three methods described in previous sections is compared. For the WOAA method nine different sets of parameters were used:

- Hamming window with steps of 32, 64, 128 and 256 samples
- Boxcar window with steps of 32, 64, 128 and 256 samples
- Boxcar window without overlap.

We compared those methods with the e_{NMAE} and e_{MSE}, by zeroing the approximation coefficients at scale 8 and the details coefficients at scale 1 in wavelet domain with mother wavelet Daubechies 8, and we concluded that the methods with steps of 32 samples had the best performance, so we use those for the following comparison.

In Tables I and II we compare the methods for the on-line processing, where we zeroed the approximation and detail

TABLE II
ERROR BETWEEN THE CLASSICAL OFF-LINE DWT PROCESSING WITH RESPECT THE METHODS ANALYZED WITH MOTHER WAVELET DAUBECHIES 8 AND ZEROING THE DETAIL COEFFICIENT IN WAVELET DOMAIN AT DECOMPOSITION LEVEL 1

	Blocks		Doppler	
	e_{MSE}	e_{NMAE}	e_{MSE}	e_{NMAE}
WOAA Hamming, $N_s = 32$	$4.76x10^{-5}$	0.0062	$7.11x10^{-7}$	0.0059
WOAA Boxcar, $N_s = N_w$	0.0142	0.2062	0.0003	0.2444
WOAA Boxcar, $N_s = 32$	0.0021	0.0417	$3.23x10^{-5}$	0.0399
WZP	0.0048	0.1314	$6.52x10^{-5}$	0.1267
WOAS	0.0076	0.1815	$2.70x10^{-5}$	0.1110

TABLE III
COMPARISON BETWEEN THE WOAA AND THE SYMMETRIC PADDING WITH MOTHER WAVELET SYMLET 4 AND ZEROING THE APPROXIMATION COEFFICIENT IN WAVELET DOMAIN AT LEVEL 8

	Blocks		Doppler		
	e_{MSE}	e_{NMAE}	e_{MSE}	e_{NMAE}	$MT(ms)$
WOAA Hamming, $N_s = 32$	0.198	0.150	0.007	0.166	0.1
Symm. pad, spin=0	0.414	0.234	0.065	0.454	4.5
Symm. pad, spin=16	0.414	0.234	0.065	0.454	66.7

coefficient respectively, with the e_{MSE} and the e_{NMAE} using as reference data **s** the off-line DWT processing with mother wavelet Daubechies 8. In those Tables we used two different signals proposed by Donoho [12]: Blocks and Doppler, which are typical test signals. We are interested at the border effect between middle blocks of the signal and we do not study the border effect at the first and last window, which will always show large distortions due to the circular convolutions involved in the DWT calculation, so those windows are ignored for the error measures. More over, we compare at first with e_{NMAE} because we are interested in eliminate the pseudo-gibbs phenomena at the border of the windows, then we compare with the e_{MSE}.

Tables I and II, show that the WOAA with Hamming window and $N_s = 32$ have the minimum e_{NMAE} for all cases, except in Table I where the Hamming window adds noise to the Blocks signal because the bandwidth of this window is larger than that of the boxcar window.

At last, in Table III, we compare the WOAA with Hamming window and $N_s = 32$ with a method for on-line processing with mother wavelet Symlet 4, with no spin and 16 spin shift, symmetric padding proposed in [13]. We used the e_{MSE}, e_{NMAE} and mean time for computing a sample (MT) for the analysis. We observed that the WOAA Hamming window method is better and faster.

IV. APPLICATION TO REAL DATA

As an application example, we have used the tool ECGsyn from the ECGtoolkit of physionet[1] to produce a realistic ECG signal sampled at 256Hz, but we modified the method to

[1] http://www.physionet.org/

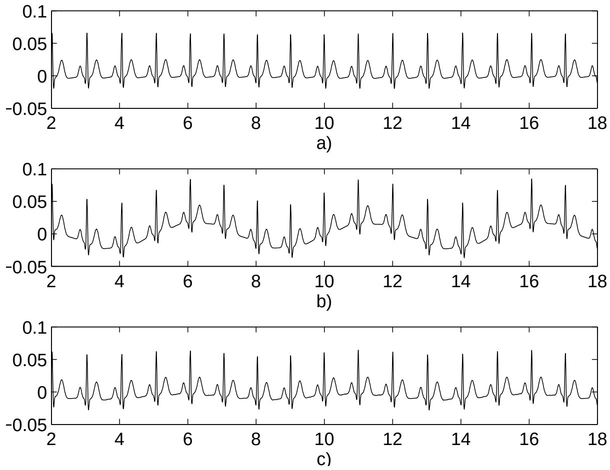

Fig. 4. a) ECG synthetic, b) ECG plus sine noise, c) Clean ECG by WOAA with hamming window, $N_s = 32$ and mother wavelet Symlet 4.

generate a signal without baseline wander. Thus we have a simulated ECG without changes in baseline, that will be our desired target signal. After that, a pure sine noise of low frequency (0.25 Hz) was added simulating the chest move noise relative to the respiration. Figure 4 shows the synthetic ECG, the ECG plus sine noise and the filtered ECG with WOAA method with the wavelet Symlet 4 at scale 8 and zeroing the approximation coefficient to obtain a cutoff frequency of approximately 0.5 Hz [14].

We have applied the method WOAA with hamming window and $N_s = 32$ to eliminate the baseline wander with mother wavelet Daubechies 8 and Symlet 4. There is no difference in errors between the original signal and the filtered one between Symlets 4 and Daubechies 8, but the first one has shorter filters and thus is faster than the second one.

As an application on real signals, a segment of the signal 111 of the MIT-BIH Arrhythmia Database was analyzed (see Fig 5), with the WOAA with Hamming window and $N_s = 32$, with a Symlet 4 at scale 8, and the approximation coefficient were fixed to zero to eliminate the baseline wander.

V. CONCLUSION

In this paper, three methods for on-line processing were adapted and tested for on-line wavelet denoising with application to ECG. However, the circular convolution of the DWT produce pseudo-gibb noise in the reconstructed signal. This artifact was analyzed in terms of objective measurement (e_{NMAE} and e_{MSE}). From the comparative study, it can be concluded that the WOAA with Hamming window and $N_s = 32$ has the best performance. This window reduce the error because is smoother in the borders than the other windows methods, which could had discontinuities at the border

This method proposed shows good performance on real data application of the elimination of the baseline wander of ECG. As this DWT can be efficiently implemented, allows to use wavelet processing in embedded systems which usually have small storage capabilities and need numerically efficient tools.

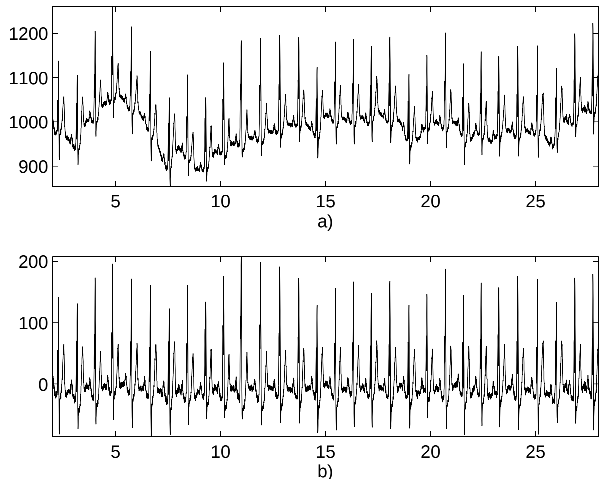

Fig. 5. a) ECG 111 of MIT-BIH with baseline wander b) ECG 111 after the proposed filter

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