ISA TRANSACTIONS®

ISA Transactions 42 (2003) 643–649

A fault detection and isolation filter for discrete linear systems

L. Giovanini,[a,*] R. Dondo[b,†]

[a]Universidad Tecnológica Nacional–Facultad Regional Villa Maria, Avenida Universidad 450, (5900) Villa María (Córdoba), Argentina
[b]Instituto de Desarrollo Tecnológico para la Industria Química (INTEC), Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)–Univ. Nac. del Litoral (UNL), Güemes 3450, (3000) Santa Fe, Argentina

(Received 2 May 2002; accepted 10 November 2002)

Abstract

The problem of fault and/or abrupt disturbances detection and isolation for discrete linear systems is analyzed in this work. A strategy for detecting and isolating faults and/or abrupt disturbances is presented. The strategy is an extension of an already existing result in the continuous time domain to the discrete domain. The resulting detection algorithm is a Kalman filter with a special structure. The filter generates a residuals vector in such a way that each element of this vector is related with one fault or disturbance. Therefore the effects of the other faults, disturbances, and measurement noises in this element are minimized. The necessary stability and convergence conditions are briefly exposed. A numerical example is also presented.

Keywords: Analytic redundancy; Directional residuals; Unbiased estimation; Uncoupling of faults

1. Introduction

Physical systems are often subjected to unexpected process dynamics that degrade the system performance. These unexpected dynamic behaviors can be classified as follows.

- Faults: nonpermitted deviation of a process characteristic that will lead to the inability for fulfilling the intended purpose. Generally they are produced by a wrong operation of instrumentation and/or control devices.
- Abrupt dynamic changes: Abnormal process development manifested as sudden parametric or structural change of the process dynamic.

In order to maintain a high level of performance, it is important that faults and abrupt disturbances be promptly detected and identified so that appropriate remedies can be applied. Model based fault detection and isolation (FDI) in dynamical systems have received growing attention in the last 30 years. In the theoretical field, over the past years several approaches to the problem of FDI in dynamical systems have been developed; for example, detection filters, the generalized likelihood ratio (GLR) method, and the multiple model method. Surveys about the matter are presented in Refs. [1–4]. Recent approaches to fault detection and isolation focus on integrating quantitative and qualitative information [5]. Applications of FDI are described in detail in Ref. [6]. The integration of FDI with process control is a relatively novel discipline called "fault tolerant control." The state of the art in this field is described in Ref. [7]. Thus clearly FDI is a very important topic from both theoretical and practical points of view and therefore it is an area of very active development.

The FDI process essentially consists of two

*Tel.: 54 (353) 437-500. *E-mail address*: lgiovani@topmail.com.ar
†Tel.: 54 (342) 455-9174/77; fax: 54 (342) 455-0944. *E-mail address*: rdondo@ceride.gov.ar

stages: residual generation and decision making. Outputs from sensors are processed to magnify the effect of a failure, if present, so that it can be recognized. The processed measurements are called residuals and the failure effect is called the signature of the failure [8]. In the absence of failures, residuals should be unbiased showing agreement between the observed and the expected normal behavior of the system. A failure signature typically takes the form of residual biases that are characteristic of the failure. Thus residuals generation is based on knowledge of the normal behavior of the system. In order to improve the fault-detection capability of a given algorithm, the generation of directional residuals is an attractive idea that was exploited in Ref. [9]. The problem of generating directional residuals was deeply analyzed from a geometric point of view in Refs. [10,11]. These works were used by Gertler [9] for developing a detection filter.

Recently, Liu and Si [12] proposed a complete order observer that is able to detect and isolate multiple faults in the continuous time domain. The gain matrix of this observer is designed in such a way that each element of the residuals vector is coupled with a given fault but uncoupled of the other faults. The algorithm works in this way only if the columns of the detectability matrix are expressed as the eigenvectors of the observer transition matrix.

The approach of Liu and Si is extended to discrete linear systems with unknown faults and/or abrupt disturbances in the present work. Although faults and abrupt disturbances represent different phenomena, they can be treated as the same behavior. Thus this work will focus only on faults. Derived results can also be applied to abrupt disturbances. The resulting algorithm presents a predictor-corrector structure similar to the structure of a standard Kalman filter. This allows us to establish the necessary conditions for stability and convergence.

This work is organized as follows: In Section 2 the fault/abrupt-disturbances detection and isolation problem is analyzed. In Section 3 the detection and isolation filter is derived from the results of Section 2. A brief stability and convergence analysis for the derived observer is presented in Section 4. In Section 5 is presented a numerical example, and finally the conclusions are outlined in Section 6.

2. Problem formulation

A discrete linear system that can present abrupt disturbances and faults can be written as follows:

$$x(k+1)=Ax(k)+B_u u(k)+Fn(k),$$
$$x(0)=x_0,\ \ u(0)=0,\ \ n(0)=0, \tag{1}$$
$$y(k)=Cx(k),$$

where $x(k)\in R^n$ is the state vector, $y(k)\in R^m$ is the measurement vector, $u(k)\in R^q$ is the control vector, $n(k)\in R^p$ is the vector of faults, and $F=[\mathbf{f}_1\mathbf{f}_2\cdots\mathbf{f}_p]\in R^{n\times p}$ is a matrix of distribution of faults. It is assumed that matrixes $A\in R^{n\times n}$, $B_u\in R^{n\times q}$, F, and $C\in R^{m\times n}$ are full-rank matrixes.

The outputs of the system at time k can be calculated from the initial-state value $x(0)$ and the effect of control actions and unknown faults over the time horizon $1\cdots k$, as follows:

$$y(k)=CA^k x(0)+\sum_{i=1}^{k-1} CA^{i-1}B_u u(k-i)$$
$$+\sum_{i=1}^{k-1} CA^{i-1}Fn(k-i). \tag{2}$$

Detectability indexes $\rho=\{\rho_1,\rho_2,\ldots,\rho_p\}$ are defined by [12]

$$\rho_i=\min\{\nu:CA^{\nu-1}\mathbf{f}_i\neq 0\quad \nu=1,2,...\},$$
$$i=1,2,...,p,$$

where $\mathbf{f}_i$, $i=1,...,p$ is the ith column of the matrix F. The index ρ_i represents the number of measurement samples that takes a fault to appear explicitly on the measurements. If the system of Eq. (1) has a finite detectability index, the matrix of faults detectability $\mathbf{D}$ [9] can be defined as [12]

$$D=\lfloor CA^{\rho_1-1}\mathbf{f}_1 \quad CA^{\rho_2-1}\mathbf{f}_2 \ \cdots \ CA^{\rho_p-1}\mathbf{f}_p\rfloor, \tag{3}$$

where $\mathbf{D}\in R^{mxp}$ and ρ_i, $i=1,...,p$ is the fault detectability index associated to $\mathbf{f}_i$. Columns of the matrix F and the elements of the vector $n(k)$ can be reordered and regrouped in accordance with the detectability index. That means that elements of the last term of the right-hand side of Eq. (2) are sorted from the largest to the smallest detectability index. Thus matrix $\mathbf{D}$ and vector $n(k)$ can be expressed as follows:

$$\overline{D}=\lfloor C\mathbf{F}_1 \;\; CA\mathbf{F}_2 \;\; \cdots \;\; CA^{s-1}\mathbf{F}_s \rfloor,$$
$$\overline{n}=[\mathbf{n}^1(k-1) \;\; \mathbf{n}^2(k-2) \;\; \cdots \;\; \mathbf{n}^s(k-s)], \tag{4}$$

where

$$\mathbf{F}_l=[\mathbf{f}_m \;\; \cdots \;\; \mathbf{f}_l] \;:\; \mathbf{f}_m \neq \mathbf{f}_l \wedge \rho_m=\rho_l;$$
$$\forall l=1,2,...,s=\max(\rho_i), \forall m=1,2,...,s=\max(\rho_i),$$
$$\mathbf{n}^l(k-l)=[n_m(k-l) \;\; \cdots \;\; n_l(k-l)];$$
$$\forall l=1,2,...,s=\max(\rho_i), \forall m=1,2,...,s=\max(\rho_i).$$

For instance, if $F=[\mathbf{f}_1\ \mathbf{f}_2\ \mathbf{f}_3]$, and $\rho_1=1$, $\rho_2=2$, $\rho_3=3$, then $\overline{D}=[Cf_1\ CA\ f_2CA^2\ f_3]$. By introducing equalities (4) into Eq. (2) and by separating the effect of ''past'' and ''present'' fault values, the system outputs can be expressed as follows:

$$y(k)=CA^k x(0)+\sum_{i=1}^{k-1} CA^{i-1}B_u u(k-i)$$
$$+\sum_{i=2}^{k-1} \overline{D}\overline{n}(k-i)+\overline{D}\overline{n}(k-1). \tag{5a}$$

It can be observed that the first two terms of the right-hand side of Eq. (5a) give the evolution of the outputs without considering faults while the other terms of the right-hand side of Eq. (5a) give the effect of past and present faults. But, under normal behavior hypothesis, the past faults and disturbances are null. Therefore under nonfault hypothesis, Eq. (5a) becomes

$$y(k)=CA^k x(0)+\sum_{i=1}^{k-1} CA^{i-1}B_u u(k-i)$$
$$+\overline{D}\overline{n}(k-1). \tag{5b}$$

Thus, under normal behavior hypothesis, the vector $y(k)$ can be re-expressed as follows:

$$y(k)=Cx(k)+\overline{D}\overline{n}(k-1). \tag{6}$$

Therefore, in expression (6), effects of faults were discriminated from effects of control actions and from the system internal dynamic. This result will be used for deriving a fault detection and isolation filter in the next section.

3. The novel filter

Let us consider a states observer of a discrete linear system:

$$\hat{x}(k+1)=A\hat{x}(k)+B_u u(k)+Kq(k), \tag{7}$$

$$\hat{y}(k)=C\hat{x}(k), \tag{8}$$

where $\hat{x}(k)$ is the estimated state vector, $\hat{y}(k)$ is the estimated measurement vector, K is the matrix gain of the observer, and $q(k)$ defines the innovation sequence or residuals sequence,

$$q(k)=y(k)-\hat{y}(k). \tag{9a}$$

In order to derive a faults and disturbances detection filter, Eq. (6) is introduced into Eq. (9a). Thus $q(k)$ can be expressed as follows:

$$q(k)=Ce(k)+\overline{D}\overline{n}(k-1), \tag{9b}$$

where $e(k)$ defines the states-estimation error:

$$e(k)=x(k)-\hat{x}(k). \tag{10}$$

It can be observed in Eq. (9b) that residuals $q(k)$ presents two different terms. The first one $[Ce(k)]$ is the states-estimation error that does not consider faults and disturbances. The second one $[\overline{D}\overline{n}(k-1)]$ is the effect of faults on the residuals. The first term contains the information necessary for correcting the states prediction while the second term biases this correction. Therefore, in order to obtain unbiased state estimations, the gain of the filter must multiply the first term while neglecting the second one. To do this, sequences $\gamma(k)$ and $q_r(k)$ are introduced:

$$\begin{bmatrix} \gamma(k) \\ q_r(k) \end{bmatrix}=\begin{bmatrix} \Sigma \\ \Pi \end{bmatrix} q(k), \tag{11}$$

where matrixes Σ and Π will be derived later. Introducing the value of $q(k)$ defined in Eq. (9b) into Eq. (11), we can obtain the following expressions:

$$\gamma(k)=\Sigma Ce(k)+\Sigma\overline{D}\overline{n}(k-1),$$
$$q_r(k)=\Pi Ce(k)+\Pi\overline{D}\overline{n}(k-1). \tag{12}$$

In order to obtain an unbiased estimation, the effects of faults and/or disturbances are to be uncoupled from the estimation error $e(k)$ but they must be used for estimating their magnitude. Therefore the following two equalities are to be verified:

$$\Sigma\overline{D}=0, \tag{13}$$

$$\Pi\overline{D}=I. \tag{14}$$

Eq. (13) is a necessary condition for an unbiased state estimation, and Eq. (14) is useful for detecting and isolating faults and/or disturbances. If Eqs. (13) and (14) are introduced into Eq. (12), this equation becomes

$$\begin{aligned}\gamma(k)&=\Sigma Ce(k),\\ q_r(k)&=\Pi Ce(k)+\overline{n}(k-1).\end{aligned} \tag{15}$$

Then, the sequence $\gamma(k)$ is to be used for correcting the predicted states, and the sequence $q_r(k)$ can be used for estimating faults and disturbances. Therefore by introducing sequences (15) into Eqs. (7) and (8) and by operating, the fault/abrupt disturbances detection filter can be written as follows:

$$\begin{aligned}\hat{x}(k+1)&=A\hat{x}(k)+B_u u(k)+\left([K \quad W]\begin{bmatrix}\Sigma\\ \Pi\end{bmatrix}\right)q(k),\\ q_r(k)&=\Pi q(k),\\ \hat{y}(k)&=C\hat{x}(k),\end{aligned} \tag{16}$$

where K is the filter gain and W is the matrix of propagation of faults and disturbances. W is defined by

$$W=A[\mathbf{F}_1 \quad A\mathbf{F}_2 \quad \cdots \quad A^{s-1}\mathbf{F}_s]. \tag{17}$$

Finally, by defining the following matrixes:

$$\begin{aligned}\mathrm{A}&=A-W\Pi C,\\ \mathrm{C}&=\Sigma C,\end{aligned} \tag{18}$$

and introducing them into Eq. (16), we can express the detection and and isolation filter as follows:

$$\begin{aligned}\hat{x}(k+1)&=[A-KC]\hat{x}(k)+B_u u(k)\\ &\quad+[K\Sigma+W\Pi]y(k),\\ q_r(k)&=\Pi q(k),\\ \hat{y}(k)&=C\hat{x}(k).\end{aligned} \tag{19}$$

Thus the observer provides state estimations $\hat{x}(k)$, measurements estimations $\hat{y}(k)$, and at the same time, faults and disturbances estimations $q_r(k)$. The remaining problem to be solved is the selection of the observer gain K but this is a standard tuning problem. If there are no system and/or measurement noises (deterministic case), the most convenient gain correspond to a deadbeat observer, but if there is some level of system and/or measurement noise (stochastic case), the most convenient gain is the Kalman gain adjusted by the noises level.

4. Stability and convergence

In this section, stability and convergence properties of the observer are briefly analyzed. The observer (19) has the structure of a Kalman filter. Stability and convergence properties of this kind of filter have been deeply analyzed in Ref. [13]. The results of this work are summarized as follows: a discrete Kalman filter is stable if and only if the eigenvalues of $[A-K(k)C]$ belong to the inner unit circle, being the couple (A, C) detectable:

$$\operatorname{rank}\left(\begin{bmatrix} zl-A\\ \mathrm{C}\end{bmatrix}\right)=n \quad \forall|z|\geqslant 1, \tag{20}$$

and the couple $(A,\mathbf{W}^{1/2})$ completely controllable:

$$\operatorname{rank}([-e^{j\omega}I+A \quad \mathbf{W}^{1/2}])=n \quad \forall\omega\in\mathbf{W}:0\leqslant\omega\leqslant 2\pi. \tag{21}$$

These conditions of stability and convergence can be extended to the filter given by Eq. (19) as follows:

$$\operatorname{rank}\left(\begin{bmatrix} zI-A & F\\ \mathrm{C} & 0\end{bmatrix}\right)=n+p \quad \forall|z|\geqslant 1, \tag{22}$$

$$\operatorname{rank}([-e^{j\omega}I+A \quad F \quad \mathbf{W}^{1/2}])=n \quad \forall\omega\in\mathbf{W}:0\leqslant\omega\leqslant 2\pi. \tag{23}$$

5. A numerical example

Let us consider the following discrete-linear system that results from the linearization and discretization of the jacketed continuous stirred-tank reactor presented in Ref. [14],

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0.8081 & -0.0006 & 0 \\ 34278 & 1.1033 & 0 \\ 0.0011 & 0.0001 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -0.0154 \\ 0.0564 \end{bmatrix} u(k) + \begin{bmatrix} 0 & 0.0226 \\ 1 & 0.4350 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} n(k) \\ d(k) \end{bmatrix},$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

The states of the system are the reactive concentration $[x_1(k)]$, the temperature into the reactor $[x_2(k)]$, and the refrigerant temperature into the reactor jacket $[x_3(k)]$. They are considered measured variables corrupted by independent "white-noises" sequences ν_1, ν_2, and ν_3 (stochastic case). The exothermic reactor is controlled by manipulating the refrigerant flow $[u(k)]$. The simulated fault is a blockade of the refrigerant control valve $[n(k)=-4,\ k\geqslant 20]$ and the simulated disturbance is a sudden change on the reactive concentration $[d(k)=0.25,\ k\geqslant 10]$. Detectability indexes defined in Ref. [12] are $\rho_n=1$ and $\rho_d=1$. The matrix $\overline{\mathbf{D}}$ defined by Eq. (4) is then

$$\overline{\mathbf{D}} = CA^{1-1}F = C[\mathbf{f}_n \quad \mathbf{f}_d] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0.0226 \\ 1 & 0.4350 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.0226 \\ 1 & 0.4350 \\ 1 & 0 \end{bmatrix}.$$

Its rank rank($\overline{\mathbf{D}}$)$=2$, thus $\overline{\mathbf{D}}$ is a full rank matrix and therefore is possible to detect and isolate non-measurable faults and abrupt disturbances. Thus matrixes Π and Σ defined by Eqs. (13) and (14) will take the following values:

$$\Pi = \begin{bmatrix} -0.0517 & 0.0027 & 1 \\ 0.2376 & 2.2865 & -2.2865 \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} 0.9971 & -0.0517 & -0.0142 \\ -0.0517 & 0.0027 & 0.0007 \\ -0.0142 & 0.0007 & 0.0002 \end{bmatrix}.$$

The chosen filter gain K corresponds to the steady state Kalman gain matrix adjusted by the known measurements-noises level. No system noises were considered. The matrix of propagation of faults is computed from its definition [Eq. (17)]. Thus they will take the following values:

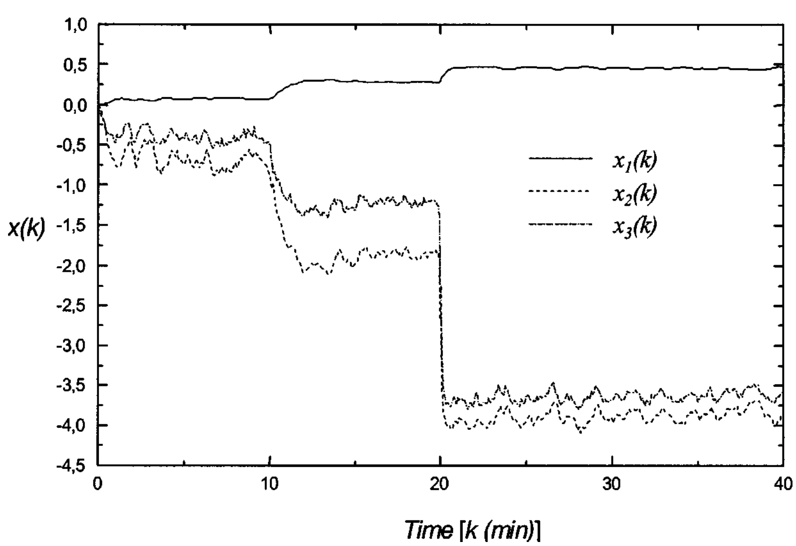

Fig. 1. Estimated states $x(k)$.

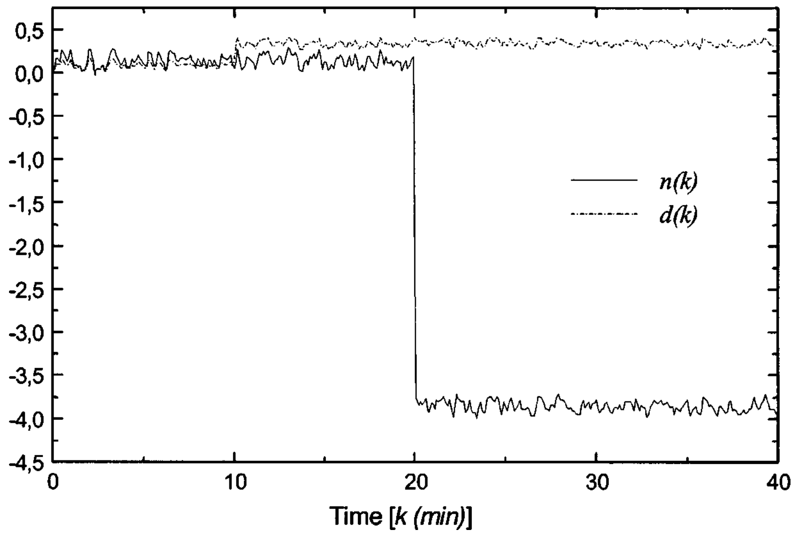

Fig. 2. Estimated faults and disturbances $q_r(k)$.

$$K=\begin{bmatrix} 0.5050 & 0.0117 & 0.0000 \\ 0.0117 & 0.9980 & 0.0000 \\ 0.0000 & 0.0000 & 0.6180 \end{bmatrix},$$

$$W=\begin{bmatrix} -0.0006 & 0.0180 \\ 1.1033 & 1.2546 \\ 1 & 0.0001 \end{bmatrix}.$$

Numerical simulations are summarized in Figs. 1 and 2. Fig. 1 shows the estimations of the system states produced by the detection filter. Note that states were expressed as deviation variables. It can be clearly observed the effects of the disturbance and of the fault on the states. Nevertheless, accurate and unbiased state estimations have been obtained. Fig. 2 shows the evolution of the estimated faults vector $q_r(k)=[\hat{n}(k),\hat{d}(k)]^T$. It can be noted that the filter showed an almost perfect isolation performance. Therefore at least in this case where every state is measured and detectability indexes are $\rho_n=1$ and $\rho_d=1$, the filter is able to detect, classify, and produce accurate diagnostics of unexpected behaviors.

6. Conclusions

In this work we have presented a fault/abrupt disturbances detection filter for discrete linear systems. The filter was designed in such a way that the directional properties of generated residuals allow uncoupling the effects of a fault/disturbance from the other faults/disturbances. The resulting algorithm is a particular Kalman filter on which each element of the residuals vector is coupled with a given fault/abrupt disturbance but is uncoupled from the remaining possible faults and disturbances of the system. Furthermore, conditions of stability and convergence for the filter have been briefly provided. A numerical example showed that the filter produced accurate state estimations and fault diagnostics.

We have presented in this brief paper an extension of the observer of Liu and Si to the discrete linear case with a perfectly known model. Nevertheless, this is an introductory work and a number of problem areas await further research. They include robustness to parametric uncertainty, tests on slowly drifted faults/disturbances (which are more difficult to detect), extension to some nonlinear systems, and tests on real systems.

References

[1] Willsky, A., A survey of design methods for failure detection in dynamic systems. Automatica **31**, 627–635 (1976).

[2] Wilsky, A., Detection of Abrupt Changes in Dynamic systems. Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, 1986.

[3] Gertler, J., Survey of model-based failure detection and isolation in complex plants. IEEE Control Syst. Mag. **8**, 3–11 (1988).

[4] Frank, P. M., Fault diagnosis in dynamic systems using analytical and knowledge base redundancy—A

survey and some new results. Automatica **26**, 459–474 (1990).
[5] Patton R., Uppal, F., and Toribio, C. Lopez, Soft computing approaches to fault diagnosis for dynamic systems: A survey. 4th IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes, Budapest, 14–16 June 2000, Vol. 1, pp. 298–311, 2000.
[6] Baseville, M. and Nikiforov, I., Detection of abrupt changes: Theory and application. Prentice-Hall, Englewood Cliffs, NJ, 1993.
[7] Patton, R., Fault tolerant control: The 1997 situation, IFAC Symposium on Fault Detection. Supervision and Safety for Technical Processes. SAFEPROCESS 97, Hull, U.K., pp. 1033–1055 (1997).
[8] Wilsky, A., Analytical redundancy and the design of robust failure detection systems. IEEE Trans. Autom. Control **29**, 603–614 (1984).
[9] Gertler, J., Generating directional residuals with dynamic parity relations. Automatica **31**, 627–635 (1995).
[10] Massoumnia, M., A geometric approach to the synthesis of failure detection filters. IEEE Trans. Autom. Control **31**, 839–846 (1986).
[11] White, J. and Speyer, J., Detection filter design: Spectral theory and algorithms. IEEE Trans. Autom. Control **32**, 593–603 (1987).
[12] Liu, B. and Si, J., Fault isolation filter design for time-invariant systems. IEEE Trans. Autom. Control **42**, 704–707 (1997).
[13] De Souza, C., Geevers, M., and Goodwin, G., Riccatti equations in optimal filtering of nonstabilizable systems having singular transition states matrices. IEEE Trans. Autom. Control **31**, 831–838 (1986).
[14] Henson, M. and Seborg, D., Nonlinear Process Control. Prentice-Hall, Englewood Cliffs, NJ, 1997.